Coexistence of Physical and Crypto Assets in a Stochastic Endogenous Growth Model

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Presentation plan

- Motivation and background
- Agents, preferences and technologies
- Generic optimization problem
- Fundamental variables
- Equilibrium
- States, controls, and policies
- Results

Motivation

- Are crypto assets a temporary phenomenon specific to current social developments?
- Are they a resource drain, a disruption or an enhancement?
- Who crowds out whom (or no one)?
- How to model crypto in a dynamic macro context?

Background

- Theory
  - Fernández-Villaverde and Sanchez (2016) – currency competition
  - Schilling and Uhlig (2019a,b) – crypto means of exchange free of policy intervention
  - (Martin and Ventura, 2018) – rational bubbles
- Empirics
  - Kristoufek (2015) – Bitcoin price drivers by investor origin
  - Cheah and Fry (2015), Cheung et al. (2015) – bubble properties of Bitcoin
  - (Rhue, 2018, Burns and Moro, 2018) ICO empirics
- Policy considerations
  - Yermack (2015), Weber (2016) – the economic nature of Bitcoin (and consorts)

Model

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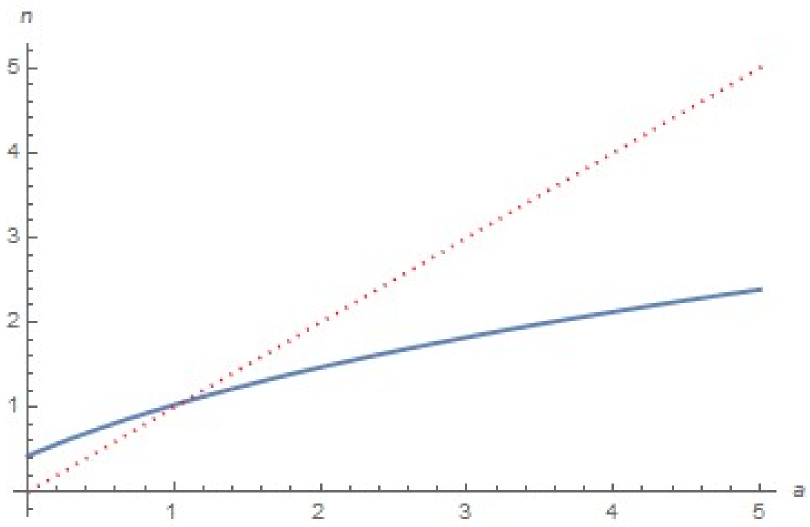
Agents

- Agents are infinitely lived, structurally identical, differ in disposable income and crypto endowments
- Each agent is a household of two: one responsible for investment, production and token purchases, the other for token sale and consumption; don‘t coordinate within the period
- Eventual crypto conversion costs are uncertain when the sale decision is taken

Leviathan-assisted absorption

- The more one earns, the bigger share must be dedicated to income protection
- Non-zero intercept: can be interpreted as UBI
- Dotted line: how this would look like without Leviathan



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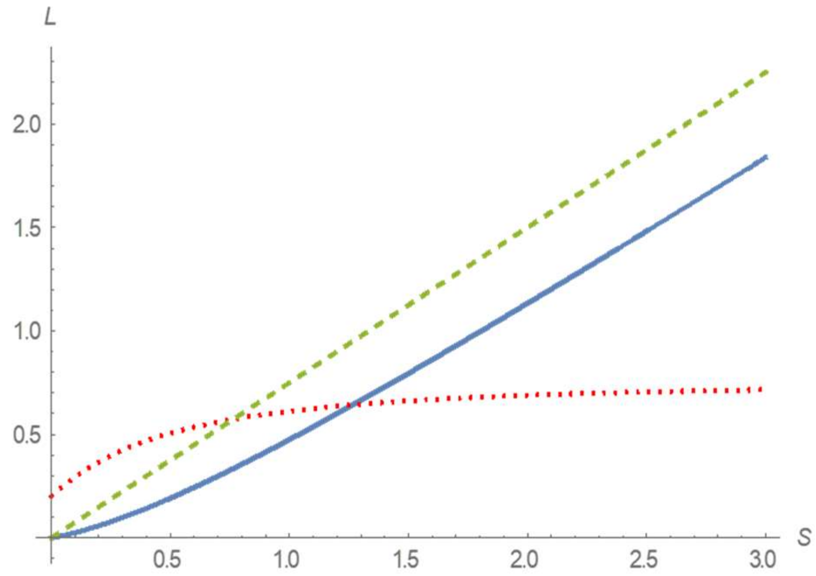
Crypto conversion

- There are exchanges allowing agents to buy and sell tokens
- There is a "gateway" token – a title to the "crypto investment fund", investment decisions inside the crypto asset ecosystem are then implicitly assumed optimal
- Back-conversion costs are non-linear, but approach linearity (with a stochastic slope) for large transaction volumes
- The featured conversion cost function is per nominal price unit (is subsequently multiplied by market-clearing price to render the sale revenue)

Conversion function:



Original variables

- Individual variables:
  - states
    - k – physical capital
    - q – output-cum-depreciated physical capital
    - x – currently owned tokens
  - controls
    - I - new physical investment
    - H – expenditure on new token purchase
    - S – back-converted tokens
- Aggregate variables
  - $\overline{K}$ - aggregate physical capital
  - $\widehat{K}$ - aggregate physical capital growth rate
  - $\bar{X}$ - total number of tokens in circulation
  - p – unit token price

Transformed variables and inter-relations

- Effective (normalized) individual states:

$$kn_t = \frac{k_t}{\bar{K}_t},\ qn_t = \frac{q_t}{\bar{K}_t},\ xn_t = \frac{x_t}{\bar{X}_t}$$

- Transformed controls:
  - b – newly purchased tokens
  - s – sold tokens as a fraction of the current state
  - $k_{t+1} = (1-\delta)k_t + I_{t+1}$ - physical capital to be used in next-period production
- Output: $y = Af(\bar{K}, k) = A\bar{K}^{1-\alpha}k^{\alpha}$
- Calculation of aggregates:
  - $\bar{K} = \int_{\Omega} k(i)\mu(di)$ - physical capital
  - $\bar{X}_t = \int_{\Omega} x_t(i)\mu(di)$ – tokens
  - $\int_{\Omega} H_t(i)\mu_t(di) = p_t \int_{\Omega} S_t(i)\mu_t(di)$ - market-clearing token price

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Transformed variables and inter-relations (cont.)

- Capital growth rate as a function of normalized states and controls:

$$\widehat{K} = \int_{\Omega} qn(i) \Big(1 - h\big(qn(i), xn(i)\big)\Big) v\big(qn(i), xn(i)\big)\mu_t(di)$$

- Normalized token price:

$$pn_t = \frac{\int_{\Omega} qn(i)h(qn(i), xn(i))\mu_t(di)}{\int_{\Omega} xn(i)s(qn(i), xn(i))\mu_t(di)}$$

- Actual vs. normalized price:

$$p_t = \frac{\overline{K}_t(1 + R_t)}{\bar{X}_t} pn_t$$

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Constraints and the objective function

- Consumption in the presence of Leviathan:

$$c = n(y - I - H) + pL(l, S)$$

- Evolution of token holdings:

$$x_t = \left(1 + R(\overline{X_{t-1}})\right)x_{t-1} - S_t + \frac{H_t}{p_t}$$

- Intertemporal utility:

$$U_t = E_t\left[\sum_{\tau=0}^{\infty} \beta^{\tau} u(c_{t+\tau})\right]$$

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Dynamics of normalized states

- Disposable income:

$$qn_{t+1} = A_{t+1}\left(\frac{qn_t(1-h(qn_t, xn_{t-1}))v(qn_t, xn_{t-1})}{\widehat{K_t}}\right)^{\alpha}$$

$$+(1-\delta)\,\frac{qn_t(1-h(qn_t, xn_{t-1}))v(qn_t, xn_{t-1})}{\widehat{K_t}}$$

- Tokens:

$$xn_t = \left(1 - \frac{s_t}{1+R_t}\right)xn_{t-1} + \frac{qn_t h_t}{(1+R_t)pn_t}$$

Maximizing utility

$$\frac{\partial U_t}{\partial b_t} = -p_t \sum_{\lambda} \pi_\lambda n'(a_t - p_t b_t) u'(c_t)$$

$$+\beta \textstyle\sum_{\lambda 1, \kappa 1} \pi_{\lambda 1} \vartheta_{\kappa 1} p_{t+1} s_{t+1} L_S(l(\lambda 1), x_t s_{t+1}) u'(c_{t+1}),$$

$$\frac{\partial U_t}{\partial s_t} = x_{t-1} p_t \sum_{\lambda} L_S(l(\lambda), x_{t-1} s_t) \pi_\lambda u'(c_t)$$

$$-x_{t-1} \beta \textstyle\sum_{\lambda 1, \kappa 1} \pi_{\lambda 1} \vartheta_{\kappa 1} p_{t+1} s_{t+1} L_S(l(\lambda 1), x_t s_{t+1}) u'(c_{t+1}),$$

$$\frac{\partial U_t}{\partial k_{t+1}} = -\sum_{\lambda} \pi_\lambda n'(a_t - p_t b_t) u'(c_t)$$

$$+\beta \sum_{\lambda 1, \kappa 1} \pi_{\lambda 1} \vartheta_{\kappa 1} (A_{\kappa 1} \alpha (\bar{K}_{t+1})^{1-\alpha} (k_{t+1})^{\alpha - 1} + 1 - \delta) n'(a_{t+1} - p_{t+1} b_{t+1}) u'(c_{t+1})$$

Model (cont.)

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Formal appearance of the solution

- There are two agent-level state variables: normalized disposable income qn (output including depreciated physical capital, divided by aggregate physical capital) and normalized crypto holdings xn (actual individually held token amount divided by their aggregate quantity in circulation)
- There are four aggregate state variables (summary statistics): physical capital stock $\bar{K}$, physical capital growth rate $\widehat{K}$, tokens in circulation X, normalized token price pn
- There is an exogenous initial asset distribution across the agent population
- There are three policy functions of state variables ($(qn, xn) \mapsto$) associated with:
  - crypto creation $h(qn,xn)$
  - crypto back-conversion $s(qn,xn)$
  - physical investment $v(qn,xn)$

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Equilibrium definition

- The equilibrium concept here is akin to the closed-loop mean-field game (MFG) equilibria of continuous-time dynamic games
- Each agent is small, i.e. unable to influence aggregate fundamentals
- Each agent employs optimal policies (as mentioned earlier), in every period taking the current values of the four aggregate states as given

Equilibrium definition (cont.)

- Evolution of the asset distribution measure is consistent with dynamic laws of motion of individual state variables (*a discrete version of the Fokker-Planck equation is involved*)
- Aggregate state variable values are consistent with individual policies, the crypto market clears
- There is balanced growth, i.e. aggregate physical capital, consumption, tokens in circulation, and the token price asymptotically grow at constant exponential rates
- In addition, an *ergodic equilibrium* is such that asset distribution is invariant under dynamic laws implied by individually optimal policies

Solution

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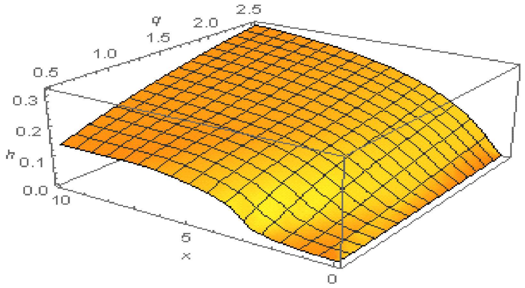
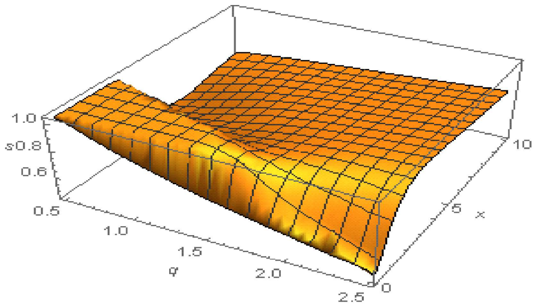
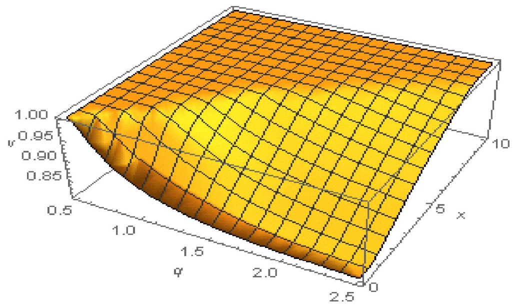
Example of the calculated optimal policy







h; new tokens are bought in the amount $\bar{K} \cdot qn \cdot h(qn, xn)$

s; tokens are converted to fiat in the amount $X \cdot xn \cdot s(qn,xn)$

v; new physical capital equals $\bar{K} \cdot qn \cdot \big(1 - h(qn, xn)\big) v(qn,xn)$

Findings

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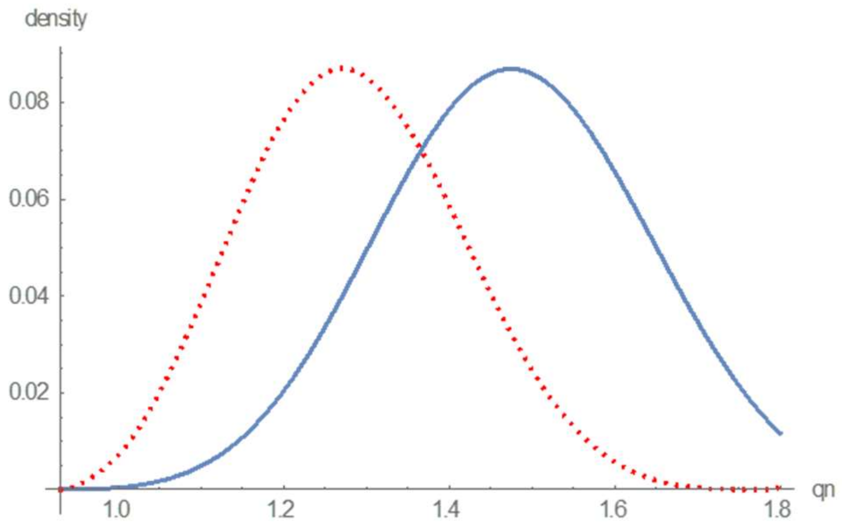
Findings

- Crypto and fiat are able of long-term coexistence as soon as one gives up the representative agent fiction
- "*Ergodically*", aggregate physical growth is higher when crypto are present
- Ergodic correlation of conventional and crypto wealth is positive
- One needs to be rich enough to want to hold crypto; the wealthiest in the society are the most enthusiastic crypto holders
- The crypto presence is a boost, but not everyone is boosted (there is a non-adoption region)
- Some agents (the "*middle class*") use conventional income to invest and crypto income to consume

Findings

Marginal physical wealth density with (solid blue line) and without (red dotted line) crypto



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Concluding caveats

- The present model lets Leviathan impair consumption, but not investment. If investment were afflicted as well, crypto would probably not be propitious for aggregate growth
- The model seems to be sensitive to the production function specification. This suggests one should pay attention to this aspect when it comes to calibrating

Thank you for your attention

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