Auctions with Tokens*

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Abstract

Motivated by the ease of creation of blockchain-based digital currencies, in this paper I allow an auctioneer to create a new token, commit to its supply, and then accept it as a mean of payment. I consider a simple, private-value auction with risk-neutral bidders, repeated twice. I show that the revenue equivalence holds, in the sense that the expected two-period revenues are the same under all common auction formats, with tokens or without tokens. However, depending on the monetary policy specified by the auctioneer, the auction with tokens differs from the one without tokens in the time profile and the variance of the auctioneer's revenues. For example, if the auctioneer commits to destroying all tokens received as payment, then he will earn with probability 1 the expected two-periods revenues at the beginning of the game. Hence, if the auctioneer is present biased or risk averse, then the auction with tokens is strictly preferred to an auction without tokens (otherwise, the two types of auctions are equivalent).

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1 Introduction

The study of auctions is among the most successful research lines in economics, having produced both deep theoretical results, and also very practical and useful in-

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sights.[1] This success was achieved by systematically studying auctions under a wide variety of assumptions with respect to bidders' valuations, information structure, equilibrium concept, etc (Klemperer, 1999). Yet, a common assumption is that all payments occur using fiat currency (for example, the USD). However, the recent invention of blockchain allows anyone to create new digital tokens and to commit to their supply.[2] These tokens can be exchanged freely on the blockchain, and can be (and often are) used as a mean of payment.

Motivated by the above observation, here I allow an auctioneer to specify an auction format, and also to create new tokens. These tokens can be used as the "internal currency" of the auction, in the sense that they are the sole mean of payment accepted by the auctioneer. I show that the revenue equivalence holds also with tokens. Nonetheless, if the auction is repeated multiple times, by issuing tokens and specifying an appropriate monetary policy, the auctioneer can manipulate the timing and variability of his earning. In some cases, he is able to eliminate all risk and earn all revenues at the beginning of the game. Hence, if the auctioneer is risk averse, the auction with tokens is preferred to the auction without tokens. Importantly, if there is perfect contracting, the auction with tokens replicates in a decentralized way the outcome of a standard auction with fiat currency in which the auctioneer and bidders also sign a side contract. However, in the presence of contracting frictions, the auction with tokens might be preferred to a standard auction with a side agreement.

Formally, I study a sequence of two private-value auctions in which two objects are sold (one per period). In every period, risk-neutral bidders draw their valuation for the object from a time-varying i.i.d. distribution. Then they submits bids. Given the profile of bids, the auction format determines the winning bidder and the payment of each player. The good is then consumed within the period. Therefore,

[1] Indeed, the 2020 Nobel prize for economics was awarded to Paul Milgrom and Robert Wilson "for improvements to auction theory and inventions of new auction formats." The Nobel committee rightly noticed that "astronomical sums of money change hands every day in auctions".

[2] There are a number internet tutorials explaining how to create blockchain based tokens (I invite the reader to search "how to create an ERC-20 Token", where ERC-20 is the simplest token that can be created on the Ethereum blockchain). There are also a number of services allowing to create a blockchain-based tokens without directly coding (see, for example, https://tokenmint.io and https://www.tokenmakerclub.com)

each auction is a very simple, static auction, repeated twice (that is, there is no connection between auctions in different periods).

The auctioneer could accept payments in fiat currency, in which case the analysis of the equilibrium is straightforward: by the revenue equivalence theorem and under an appropriate assumption on the distribution of valuations, all common auction formats with a reservation price of zero maximize revenues. It follows that, in each period, the largest revenues the auctioneer can earn are equal, in expected value, to the expected second-highest valuation. If the auctioneer is risk averse, however, the variance of the revenues will also affect his utility.

But the auctioneer could also create a new blockchain-based token. I consider an auction with tokens which is as close as possible to the auction without tokens: bids and payments are expressed in fiat currency (for example USD), but need to be settled using the token created by the auctioneer. More precisely, in the auction with tokens after bids are submitted, the winner is announced together with a profile of payments that depends on the auction format and the profile of bids. At this point, a market for tokens opens in which bidders and the auctioneer exchange tokens. The bidders then the use the tokens acquired to pay the auctioneer.

The key insight is that, in an auction with tokens, in period 1 tokens might be purchased for speculation and not for bidding. This will happen when the realized valuations in period-1 are low (relative to the period-2 expected valuations). In this case the demand for tokens *for bidding* in period 1 is low. This creates an arbitrage opportunity for bidders: they may purchase tokens in period 1 for then selling them in period 2. This arbitrage opportunity implies that, for any specific realization of the period-1 valuations, the equilibrium price for tokens in period 1 cannot be lower than the expected period-2 price of tokens.

The possibility of speculating with tokens therefore creates a lower bound to the period-1 price for tokens. As a consequence, the auctioneer's period-1 revenues are higher in an auction with tokens than in an auction without tokens. At the same time, in period-2, the auctioneer may face competition from bidders on the market for tokens—in the sense that, to the extent that bidders bought tokens that they did not use for bidding, they will also be selling tokens alongside the auctioneer. Hence, the auctioneer's period-2 revenues are *lower* in an auction with tokens than in an auction without tokens. Furthermore, the speculative demand for tokens is deterministic, as it depends on the expected period-2 valuations and not on the realized period-1 valuations. The auction with tokens therefore differs from the auction without tokens both in the time profile and in the variance of the auctioneer's earnings.

Interestingly, here the specific monetary policy (i.e., the rules determining the creation of destruction of tokens between periods) can be considered as part of the auction format. With this respect, I study two types of monetary policy. The first one is a monetary policy that increases (or shrinks) all tokens by a given factor. I show such monetary policy is irrelevant, in the sense that it does not affect neither the total two-period revenues earned (which are equal to those in the auction without tokens), not the auctioneer's utility. I then consider a monetary policy inspired by staking, in which the tokens used for bidding grow (or shrink) at a different rate than tokens used not for bidding.[3] Again, this policy does not affect the total two-period revenues. Nonetheless, it affects the auctioneers' utility. To see this, consider an extreme form of this policy: one that destroys all tokens used for bidding (or give an infinitely large reward to those not used for bidding). Under this policy, the entire two period revenues are earned by the auctioneer at the beginning of the game. Furthermore, if trading of tokens is allowed also before valuations are drawn, then these revenues are earned with probability 1.

It is therefore possible to create an auction with tokens that removes all risk and also front-load all payments to the auctioneer. If the auctioneer can save (so that these earnings can be optimally reallocated to different periods), then the auction with tokens is preferred to the auction without tokens, strictly so if the auctioneer is risk averse. Interesting, the benefit of an auction with tokens (relative to one without tokens) increases in the presence of borrowing constraints. The reason is that, in an auction without tokens, achieving the optimal consumption smoothing may require the auctioneer to borrow. Hence, if borrowing is constrained, then optimal consumption smoothing may not be possible. In an auction with tokens, all revenues accrue at the beginning. Hence, the auctioneer can achieve the optimal

[3] Staking generally refers to the allocation of additional tokens to those who "lock" or otherwise do not use their tokens. Here tokens are used for bidding, so staking refers to the differential rewards received by tokens that have been used for bidding relative to those that were not used.

consumption smoothing by only saving.

Because the auctioneer is creating a new currency, a crucial assumption of the model is that the auctioneer can commit to a specific monetary policy, which can be achieved by regulating the supply of the token via a blockchain-based smart contract. It is also important that the auctioneer can commit to the auction format, in the sense that the auctioneer cannot refuse to accept tokens as a means of payment after announcing that he would do so. Again, depending on the nature of the object sold, this type of commitment could be achieved via a smart contract. Finally, I do *not* assume that the auctioneer can commit to behaving in a certain way on the market for tokens—for example, when selling tokens the auctioneer cannot commit to buying them back at a later date.

Literature review

The mechanism design literature noticed long ago that certain centralized mechanism can be implemented in a decentralized way by using tokens (see, for example, Ostroy and Starr, 1974, Kocherlakota, 1998). Similarly, some of the early papers in monetary theory considered general-equilibrium models in which there is at least an equilibrium in which money emerges (see Samuelson, 1958, and Townsend, 1980). Because the equilibrium with money is Pareto superior to that without money, again, we can think of money as allowing for the decentralized implementation of some (usually constrained) optimal allocation. More closely related are models in which money has value because of exogenous reasons. For example, in Starr (1974), a government creates money and establishes that taxes need to be paid using money. Similarly to what happen in our model, in Starr (1974), money has strictly positive value also in a finite-horizon model. Of course, the fact that the government creates money and gives it value opens the issue of monetary commitment (see, for example, Lucas Jr and Stokey, 1983).

The advent of blockchain and blockchain-based tokens provided new impulse to the above literature (for an overview, see Townsend, 2020). The reason is that, thanks to blockchain, anyone can create tokens at (almost) no cost. Also, smart contracts can be used to generate commitment, for example to a given monetary policy, or to perform payments based on contingencies. Several authors have therefore studied how blockchain-based tokens and smart contracts can be used to implement various types of mechanism in a decentralized way (see, Holden and Malani, 2019, Gans, 2019, Lee, Martin, and Townsend, 2021).

With this respect, note that, in the model presented here, the auction itself could be a traditional, centralized auction, or a decentralized one (via a smart contract). Also, auctions can be held with or without tokens (and, in each case, with or without a smart contract). Hence, tokens do not affect the implementation of the mechanism (i.e., the auction). Tokens however allow for the decentralized implementation of a risk-sharing agreement between bidders and the auctioneer. It follows that, in a world of perfect contracting, issuing tokens and signing a contractual agreement will achieve the same outcome. If, however, there are contracting frictions (for example, certain relevant variables are observable but not contractible) then issuing tokens may be more efficient that a contractual agreement.

A number of papers studied theoretically firms' incentives to issue blockchain-based tokens, which either represent a pre-sale of a given unit of future output, or the only currency that the firm will accept in the future. Some of these papers showed that, in the presence of network externalities, selling tokens helps avoiding coordination failures (Sockin and Xiong, 2018, Cong et al., 2021, Bakos and Halaburda, 2018, and Li and Mann, 2018). Other papers focused on the sale of tokens as an innovative way to raise capital and finance the development of a product or a platform (Catalini and Gans, 2018, Malinova and Park, 2018, Canidio, 2018, Bakos and Halaburda, 2019, Goldstein et al., 2019, Cong et al., 2020, Canidio, 2020, Gryglewicz et al., 2021, Garratt and van Oordt, 2021, Chod and Lyandres, 2021). In the model considered here, the auctioneer has no financing need, the valuations for the product are exogenously given, and there are no network externalities. Hence, tokens are sold purely to earn a profit. Nonetheless, there is a connection with the above literature, because I show that issuing tokens and implementing an appropriate monetary policy allows the auctioneer to manipulate his time-profile of earnings and the risk faced by the auctioneer. Both these elements are important in determining the incentives to invest and create new ventures.

I will frequently refer to two important results in auction theory. The first is the revenue equivalence theorem, which states:[4]

> *Assume each of a given number of risk-neutral potential buyers of an object has a privately-known signal independently drawn from a common, strictly-increasing, atomless distribution. Then any auction mechanism in which (i) the object always goes to the buyer with the highest signal, and (ii) any bidder with the lowest-feasible signal expects zero surplus, yields the same expected revenue (and results in each bidder making the same expected payment as a function of her signal).*

For our purposes, the above statement implies that all common auction formats (i.e., first-price, second-price, all-pay, ...) generate the same expected payment from bidders and hence the same expected revenues to the auctioneer. The second result is the design of optimal auctions.[5] In particular, in the model I will assume that the distribution of valuations is such that all common auction formats with a reservation price of zero maximize the auctioneer's expected revenues.

2 The model

I consider a single-object private-value auction, repeated twice. There are $n \geq 2$ ex-ante identical bidders and an auctioneer. In period 0, the auctioneer decides the auction format. If the auction format requires the use of tokens, then the auctioneer creates an initial stock of tokens M, and also announces a monetary policy, that is, how the stock of tokens will evolve over time (see below). Then, in every period $t \in \{1, 2\}$ the auctioneer sells a single object according to the auction format specified initially. Each object sold has zero value to the auctioneer.

There is no discounting. Bidders are risk neutral and cash abundant, in the sense that their cash constraint is never binding. The auctioneer has a time-varying concave utility function $U_t()$, for $t \in \{1, 2\}$. For the time being, I assume that the

[4] Vickrey (1961) developed some special case of the revenue equivalence theorem. The statement presented here is taken from Klemperer (1999), and summarizes results in Myerson (1981) and Riley and Samuelson (1981). For a more general formulation, see Milgrom and Segal (2002).

[5] See, again, Myerson (1981), Bulow and Roberts (1989), Bulow and Klemperer (1996) and Klemperer (1999).

auctioneer cannot borrow nor save, and hence must consume in each period the totality of the revenues raised in that period. I relax this assumption in Section 4.1.

Auctions without tokens If the auctioneer runs an auction that does not require tokens, then in each period $t \in \{1, 2\}$:

- First, each bidder draws a valuation $v_{i,t} > 0$ from a continuous and atomless distribution with c.d.f $F_t(v)$, p.d.f. $f_t(v)$ and support $[\underline{v}, \overline{v}]$. The auction is in private values, and hence each bidder's valuation is independent of the other bidders' valuations. Each $v_{i,t}$ is bidder i's private information, but the distributions $F_t(v)$ for $t \in \{1, 2\}$ are common knowledge.

- Then, each bidder sends a message $b_{i,t} \in \mathbb{R}_+$ to the auctioneer, interpreted as his bid.

- As a function of the messages received and the auction format initially announced, the auctioneer determines who is the winner and a payment $\beta_{i,t} \leq b_{i,t}$ for each bidder (implicitly a function of all messages received).

- The winning bidder enjoys a payoff equal to $v_{i,t} - \beta_{i,t}$; all other bidders enjoy a payoff equal to $-\beta_{i,t}$; the auctioneer enjoys a per-period payoff equal to $U_t(\sum_i^n \beta_{i,t})$.

Finally, to avoid uninteresting complications, I assume that $vf_t(v) \geq 1 - F_t(v)$ for all $t \in \{1, 2\}$ and $v \in [\underline{v}, \overline{v}]$. As we will see , under this assumption, any standard auction format with a reservation price of zero maximizes the auctioneers' revenues, and calculating the revenues from the optimal auction without tokens is straightforward.

Auctions with tokens If the auctioneer uses tokens, then the timeline of each period $t \in \{1, 2\}$ is the following:

- Again, at the start of a period, each bidder draws a valuation $v_{i,t}$ from the distribution $F_t(v)$. Note that, at this point, both auctioneer and bidders may own tokens that they accumulated from previous periods. Call $A_t \geq 0$ the tokens owned by the auctioneer at the beginning of the period, and $a_{i,t} \geq 0$ the tokens owned by bidder i. By assumption, $A_1 = M$ and $a_{i,1} = 0$ for all $i \leq n$.

- Then, each bidder sends a message $b_{i,t} \in \mathbb{R}_+$ to the auctioneer, interpreted as his bid *in USD*.

- As a function of the messages received and the auction format initially announced, the auctioneer determines who is the winner and a payment $\beta_{i,t} \leq b_{i,t}$ for each bidder (implicitly a function of all messages received). This payment is expressed in USD, *but needs to be settled using tokens*.

- A frictionless, anonymous financial market for tokens opens, in which both the auctioneer and bidders participate. All market participants are price takers, and have deep-pockets in the sense that there is no upper bound to the amount they can spend in purchasing tokens.
  Call p_t the equilibrium price for tokens; call $q_{i,t}$ the equilibrium demand for tokens of bidder i; call Q_t the equilibrium demand for tokens of the auctioneer. Feasibility implies $A_t + \sum_i a_{i,t} = Q_t + \sum_i q_{i,t}$. Finally, because tokens will be used to pay the auctioneer (see the next point) it must be that $q_{i,t} \geq \frac{\beta_{i,t}}{p_t} - a_{i,t}$.

- Then, each bidder sends $\frac{\beta_{i,t}}{p_t}$ tokens to the auctioneer. At this point, bidders own $a_{i,t} + q_{i,t} - \frac{\beta_{i,t}}{p_t}$ tokens, and the auctioneer owns $A_t + Q_t + \sum_i \frac{\beta_{i,t}}{p_t}$ tokens.

- The winning bidder enjoys a per-period payoff equal to the value of the object minus the expenditure in tokens, that is: $v_i - p_t \cdot q_{i,t}$.[6] Similarly, the losing bidders enjoy a per-period payoff equal to $-p_t q_{i,t}$, and the auctioneer enjoys a per-period payoff equal to $U_t(-p_t Q_t)$.

- The stock of tokens changes according to the monetary policy announced by the auctioneer. Here I consider two possible monetary policy parameters: a uniform increase (or decrease) of all tokens by the same factor $\tau < -1$, and an increase (or decrease) of only the tokens used for bidding by a factor $\sigma < -1$. As a result at the beginning of the subsequent period each bidder

[6] Note that $q_{i,t}$ could be negative, in which case the bidder earned money by selling tokens

i owns $a_{i,t+1} = (1+\tau)(a_{i,t} + q_{i,t} - \frac{\beta_{i,t}}{p_t})$, while the auctioneer owns $A_{t+1} = (1+\tau)(A_t + Q_t + (1+\sigma)\sum_i \frac{\beta_{i,t}}{p_t})$.

Finally, the auction formal is assumed to be such that: (i) those bidding the most tokens win, (ii) those bidding zero tokens pay zero and win with probability zero.[7]

Note that the monetary policy considered here may sound far fetched, but is actually very simple to implement in the context of blockchain based tokens. In particular, the fact that the tokens used for bidding may grow at a different rate than the other tokens is inspired by staking. In staking, those who "lock" some tokens (or, more in general, do not use them) are rewarded with additional tokens. Here the staking reward is positive if $\sigma < 0$, in which case those who do not use the tokens receive an additional reward relative to those who do use them.

The above auction with tokens is very similar to an auction without tokens because bids and payments are expressed in USD. The only difference is that those payments need to be settled using the token—which is therefore a mean of payment but not a unit of account. However, other assumptions are possible. For example, bidders may be required to bid by submitting tokens, which could then be partially returned to the bidders after the winner is determined.[8] Perhaps more interesting given the context, here the monetary policy is part of the auction format. Here I consider only two possible policies, but many more are possible. The bottom line is that the above is the least complex auction with tokens, and not the most general auction with tokens.

Note also that, like in the auction without tokens, the winner can consume the object at the end of the period. However, tokens cannot be consumed directly, but need to be exchanged for USD in the following period. This implies that, in period 1, the cost of purchasing tokens affects negatively bidders' utility. However, in period 2, the bidders can sell some of the tokens accumulated (and only keep the ones they need to bid) and consume the revenues earned.

[7] Hence, if this was an auction without tokens, the revenue equivalence theorem would hold.

[8] In this case, there is an additional complication: bidders need to purchase tokens before bidding, which means that the equilibrium price for tokens may reveal some information relative to the realized distribution of valuations. Hence, the equilibrium on the market for tokens is a rational expectation equilibrium.

3 Solution: auction without tokens

If the auction is without tokens, then all the standard results from auction theory apply.[9] Quite immediately, in every period, the revenue equivalence theorem holds: all standard auction formats generate the same expected revenues. Also, a second price auction with a reservation price of zero maximizes the auctioneers revenues, which are

$$k_t \equiv E_t[v_{Max-1,t}],$$

where $v_{Max,t} \equiv \max_i\{v_{i,t}\}$ is the realized highest valuation in period t, and $v_{Max-1,t} \equiv \max_{i \neq Max}\{v_{i,t}\}$ is the realized second-highest valuation in period t. Hence, if the auctioneer decides not to use tokens, the maximum revenues he can earn are:

$$\Pi_{USD} = k_1 + k_2$$

For a given auction format, the auctioneer's utility is

$$U_{USD} = EU_1(\sum_i \beta_{i,1}) + EU_2(\sum_i \beta_{i,2}).$$

Hence, if the auctioneer is risk averse (i.e. his utility function is strictly concave in at least one period), not all common auction formats maximize expected utility. The reason is that the variance of the total payment received by the auctioneer also matters for his utility. Nonetheless, in what follows, it is sufficient to note that for any auction format, it must be that

$$U_{USD} \leq U_1(k_1) + U_2(k_2)$$

with strict inequality if the auctioneer is risk averse.

[9] See, for example, Klemperer (1999), in particular Section 4 (for the revenue equivalence theorem) and Appendix B (how to calculate the optimal reservation price).

4 Solution: auction with tokens

Consider the last period of the game. Clearly, tokens have no continuation value. For this reason, bidders will purchase the strictly minimum amount of tokens to bid, so that $\beta_{i,2} = p_2(a_{i,2} + q_{i,2})$. Hence, for given profile of bids, bidder i's payoff in period 2 is

$$\begin{cases} v_{i,2} + p_2 a_{i,2} - \beta_{i,2} & \text{if } b_{i,2} > \max_{j \neq i} \{b_{j,2}\} \\ p_2 a_{i,2} - \beta_{i,2} & \text{otherwise,} \end{cases} \tag{1}$$

Also, the auctioneer will sell all his tokens on the market, which implies that

$$p_2 = \frac{\sum \beta_{i,2}}{(1+\tau)\left(M + \sigma \frac{\sum_i \beta_{i,1}}{p_1}\right)}$$

where $(1+\tau)\left(M + \sigma \frac{\sum_i \beta_{i,1}}{p_1}\right)$ is the total stock of tokens in period 2.

Consider now the bidders' problem of how to bid. Note that this problem is identical to that of an *all-pay auction without tokens*. By the revenue equivalence theorem, expected revenues in this all-pay auction (without tokens) are equal to that of a second-price auction (without tokens), which implies

$$E[\sum \beta_{i,2}] = k_2.$$

Hence, at the beginning of period 2, the expected period-2 price for tokens is

$$p_2^e = \frac{k_2}{(1+\tau)\left(M + \sigma \frac{\sum_i \beta_{i,1}}{p_1}\right)},$$

For future references, note that whereas the incentives to bid in period 2 are the same as in an all-pay auction with USD, the players' payoffs need not to be. By Equation 1, the players' payoffs depend both on the outcome of the auction, and on the initial distribution of tokens: before the valuations for period 2 are drawn, bidder i expects to earn $w_2 \equiv E_t[\max\{v_i - v_t^{Max-1}, 0\}]$ in the auction, and $p_2 a_{i,2}$

from the sale of tokens.[10] Each bidder i's expected period-2 payoff is therefore

$$u_2(a_{i,2}) \equiv p_2^e a_{i,2} + w_2.$$

Similarly, the auctioneer expected period-2 payoff is

$$U_2(A_2) \equiv p_2^e A_2.$$

An interesting corollary is that the auctioneer's period-2 payoff is the same as in the auction without tokens if and only if the auctioneer owns all the tokens and will be strictly lower otherwise.[11]

Consider now period 1. The next lemma derives the bidder's payoff and equilibrium price of tokens for given profile of bids.

Lemma 1. *For given realization of valuations and given profile of bids $b_{i,1}, ..., b_{n,1}$, bidder $i \in \{1, ..., n\}$'s payoff is*

$$\begin{cases} v_i - \beta_{i,1} + w_t & \text{if } b_{i,1} > b_{j \neq i} \\ -\beta_{i,1} + w_t & \text{otherwise.} \end{cases}$$

The equilibrium demand for tokens is

$$\sum_i^n \beta_{i,1} + S$$

where

$$S = M\left(\frac{\max\left\{k_2 - (1+\sigma)\sum_i \beta_{i,1}, 0\right\}}{k_2 - \sigma \sum_i \beta_{i,1}}\right)$$

is the speculative demand for tokens, that is, the demand for tokens not used for

[10] It is useful to think of each bidder selling all their tokens and earning $p_2 a_{i,2}$, while simultaneously purchasing tokens to bid $\tilde{\beta}_{i,2}$. The expected cost of the bid is part of the expected payoff from the auction.

[11] This also implies that, if there is only one period, then an auction with tokens is identical to an auction without tokens.

paying the auctioneer. The equilibrium period-1 price is

$$p_1 = \frac{\max\{\sum_i^n \beta_{i,1}, k_2 - \sigma \sum_i \beta_{i,1}\}}{M}.$$

The most important observation is that, in period 1, bidders may purchase tokens and not use them for bidding. This will happen in equilibrium when the realized distribution of valuations is such that total payments to the auctioneer are low. In this case, if the demand for tokens was determined exclusively by the tokens used for bidding, the price for tokens would be lower (in expectation) in period 1 than in period 2, which cannot be an equilibrium. The fact that there can be a speculative demand for tokens, in turns, implies that the period-1 price for tokens has a lower bound: it cannot be lower than $(1+\tau)p_2^e$.

With respect to the bidder's payoff, when $p_1 > (1+\tau)p_2^e$ the price for tokens is decreasing over time and hence bidder will liquidate all tokens but the ones they need to bid in period 1. If instead $p_1 = (1+\tau)p_2^e$, bidders may carry some tokens to period 2, but their value is the same in both periods, so the bidder's payoff does not depend on how many tokens are carried to the next period. In either case, the bidder's problem is identical to that of a standard auction without tokens. By the revenue equivalence, $E[\sum_i \beta_{i,1}]$ is the same in all standard auction formats.

Knowing this, I can compute the auctioneer's revenues. To start, note that in equilibrium the speculative demand for tokens is held by bidders. The reason is that bidders are indifferent between holding any amount of tokens for speculative purposes, while if the auctioneer is risk averse, he will strictly prefer not to hold tokens for speculative reasons.[12] Hence, the auctioneer's stock of tokens at the beginning of period 2 is:

$$(1+\sigma)(1+\tau)\frac{\sum_i^n \beta_{i,1}}{p_1} = (1+\sigma)(1+\tau)M\frac{\min\{\sum_i^n \beta_{i,1}, k_2 - \sigma \sum_i \beta_{i,1}\}}{k_2 - \sigma \sum_i \beta_{i,1}}$$

[12] If the auctioneer is risk neutral, then assuming that bidders hold all the speculative demand for tokens is without loss of generality, because also the auctioneer would be indifferent between holding any amount of speculative demand.

Hence, the auctioneer's two-periods expected revenues are

$$\begin{aligned}\Pi_{tokens} &= p_1^e M + E\left[p_2^e(1+\sigma)(1+\tau)M\frac{\min\{\sum_i^n \beta_{i,1}, k_2 - \sigma\sum_i \beta_{i,1}\}}{k_2 - \sigma\sum_i \beta_{i,1}}\right] \\ &= E\left[\max\{\sum_i^n \beta_{i,1}, k_2 - \sigma\sum_i \beta_{i,1}\}\right] + E\left[\min\{(1+\sigma)\sum_i^n \beta_{i,1}, k_2\}\right] = k_1 + k_2.\end{aligned}$$

Which is the same as in the auction without tokens. I summarize these results in the following proposition.

Proposition 1. *If the monetary policy is uniform, then the auctioneer two-periods revenues are equal to the two-period revenues of the auction without tokens for all* $M > 0$, $\tau > -1$, *and* $\sigma > -1$.

Proof. In the text. □

As previously discussed, the fact that tokens can be bought for speculative reasons generates costs and benefits to the auctioneer. In period 1, there is a benefit because the period-1 price is now bounded below. At the same time, the auctioneer's receive fewer tokens back and hence starts period 2 being poorer then if the speculative demand was absent. The above proposition shows that, for all possible τ, σ and M these two effects cancel out.

This emerges clearly from the auctioneer's utility, which is

$$\begin{aligned}U_{tokens} &= EU_1(p_1 M) + EU_2\left[p_2(1+\sigma)(1+\tau)M\frac{\min\{\sum_i^n \beta_{i,1}, k_2 - \sigma\sum_i \beta_{i,1}\}}{k_2 - \sigma\sum_i \beta_{i,1}}\right] \\ &= EU_1\left[\max\{\sum_i^n \beta_{i,1}, k_2 - \sigma\sum_i \beta_{i,1}\}\right] + EU_2\left[\frac{(1+\sigma)\sum_i \beta_{i,2}}{\sigma + \max\{\sum_i \beta_{i,1}, k_2 - \sigma\sum_i \beta_{i,1}\}}\right]\end{aligned}$$

For intuition, consider the case $\sigma = 0$. In this case the auctioneer's utility is

$$\begin{aligned}U_{tokens,\sigma=0} &= EU_1(p_1 M) + EU_2\left[p_2 M(1+\tau)\frac{\min\{\sum_i^n \beta_{i,1}, k_2\}}{k_2}\right] \\ &= \mathrm{pr}\{\sum \beta_{i,1} > k_2\}\left(EU_1(\sum \beta_{i,1} | \beta_{i,1} > k_2) + EU_2(\sum \beta_{i,2})\right) + \\ &\quad \mathrm{pr}\{\sum \beta_{i,1} < k_2\}\left(U_1(k_2) + EU_2\left(\frac{\sum \beta_{i,2} \cdot \sum \beta_{i,1}}{k_2} | \sum \beta_{i,1} < k_2\right)\right)\end{aligned}$$

Hence, conditional on $\sum_i^n \beta_{i,1} > k_2$, utility is the same as in the auction without tokens. However, conditional on $\sum b_{i,1} < k_2$, there are important differences between this case and the one without tokens. To start, period-1 expected revenues in the auction with tokens are equal to the period-2 expected revenues in the auction without tokens, and the same holds for period 2 revenues. Furthermore, in period 1 the auctioneer faces no risk, while in period-2 the auctioneer faces additional risk.

Note also that neither M nor τ affect the speculative demand for tokens, and hence neither M nor τ affect period-1 and period-2 revenues. The reason is that, if τ or M change, the price of tokens in period 2 change, but speculator's are fully compensated by an equal increase in their stock of tokens. The parameter σ instead matters for the speculative demand for tokens, and hence for how total revenues are distributed between the two periods. To see how this reflects on the auctioneer's utility, consider two extreme cases:

- If $\sigma > 0$ and sufficiently large, then it is easy to see that the auctioneer's utility is equal to that of the auction without tokens. This is quite intuitive: if $\sigma > 0$ and sufficiently large, then there is never any speculative demand for tokens. The entire stock of tokens is sold by the auctioneer in period 1, then received back as payment, then sold again in period 2.

- $\sigma = -1$, we have that $U_{Tokens,\sigma} = EU_1(\sum \beta_{i,1} + k_2)$. In this case the auctioneer destroys the tokens he receives as payments in period 1. The result is that period-2 payoff is zero, and the entire revenues are earned in period 1. Interestingly, the auctioneer earns period-2 expected revenues for sure in period 1.

Whether the case $\sigma = -1$ generates higher or lower utility than the auction with tokens is ambiguous. In an auction with tokens with $\sigma = -1$ there is less risk, but the revenues are earned only in period 1. In an auction with tokens there is more risk, but consumption is spread over 2 periods. Whether allowing for risk but also for consumption smoothing (as in the auction without tokens) is better or worse than reducing risk but also removing consumption smoothing (as in the auction with tokens) will depends on the shape of the utility function.

The comparison between the two cases becomes unambiguous if the auctioneer can also save, as the next section shows.

4.1 Savings and borrowing

Suppose the auctioneer can save. In case of an auction with tokens with $\sigma = -1$, then the auctioneer earns $\beta_1 + k_2$ in period 1, consume some of this in period 1 and some in period 2. This way, he can achieve optimal consumption smoothing.

The possibility of saving also affects the utility from holding an auction without tokens. Nonetheless, for all possible levels of savings, the auctioneer will still need to bear some risk in period 2, which is not the case in an auction with tokens. As a consequence, his utility will be lower than in the auction with tokens with $\sigma = -1$.

I summarize these observations in the following proposition.

Proposition 2. *If the auctioneer can save and is risk averse, then the auction with tokens with $\sigma = -1$ is strictly preferred to an auction without tokens.*

Finally, an interesting corollary having to do with the ability to borrow. In case the auctioneer holds an auction with tokens with $\sigma = -1$, he can then optimally smooth consumption by savings. In case of an auction without tokens, instead, optimal consumption smoothing may require the auctioneer to borrow. Hence, the presence of borrowing constraint or other types of credit market frictions increases the benefit of holding an auction with tokens relative to one without tokens.

4.2 Pre-sale of tokens

I now extend the model by allowing the trading of tokens already in period 0, before the start of the auction. Clearly, in period 0 the only demand for tokens is a speculative demand, which implies that

$$p_0 = E[p_1]$$

It is easy to check that, if the auctioneer runs an auction with tokens with $\sigma = -1$, he can sell all his tokens in period 0 and earn $k_1 + k_2$ with probability 1. By saving what he earned, he can then consume a fraction of his earnings in period

1 and the rest in period 2. Hence, the pre-sale of tokens together with the ability to save allows the auctioneer to eliminate all risk, while at the same time achieve the optimal consumption profile.

Remember that, here, any common auction format with a reservation price of zero maximizes revues. The above discussion therefore implies the following proposition.

Proposition 3. *If the auctioneer can save and there is a pre-sale of tokens, then any common auction format that*

- *has a reservation price of zero*
- *uses tokens*
- *implements a monetary policy* $\sigma = -1$

is optimal.

5 Discussion: contractual agreement

I showed that, if the auctioneer can save, he can design an auction with tokens that eliminate all risks and achieve optimal consumption smoothing. Because bidders are purchasing tokens, they are effectively bearing all the risk and also front-loading all payments to the auctioneer. This is efficient because bidders are assumed risk neutral and cash abundant. The important observation is that, if the auctioneer and the bidders could write a side contract, bidders would fully insure the auctioneer, and also borrow/lend money to the auctioneer so that he can achieve optimal consumption smoothing. Hence, issuing tokens and writing a contract can, in principle, achieve the same outcome.

There are however two important considerations. First, although a contract may achieve the same outcome than issuing tokens, they may not generate the same utility for the auctioneer. Clearly, in the case of an auction with tokens, the entire surplus generated by removing risk and implementing the optimal consumption smoothing is earned by the auctioneer. In the case of contracting, this surplus will be shared as a function of the specific bargaining protocol or bargaining solution considered.

Second, there could be a number of contracting frictions. Note that when issuing tokens, the bidders collectively insure the auctioneer from all risks. Doing this via a contract therefore requires coordinating multiple parties. Also, if certain elements may be observable but not contractible (for example, to the two distributions of valuations), then it may not be possible to achieve an efficient outcome via a contract. Finally, as already discussed in the introduction, creating tokens and trading tokens can be done at almost no cost, while writing and enforcing contracts may generate significant costs.

To summarize, in a world in which contracts are perfect and costless, then issuing tokens achieves the same outcome than a contractual agreement, but in a decentralized way. However, in the presence of any contracting frictions or costs, then issuing tokens is strictly preferred to writing a contract.

A Mathematical derivations

Proof of Lemma 1. For given realization of the bidder's valuations and given bids, there are two possible cases: $p_1 > (1+\tau)p_2^e$ or $p_1 = (1+\tau)p_2^e$.[13]

If $p_1 = (1+\tau)p_2^e = \frac{k_2}{M+\sigma\frac{\sum \beta_{i,1}}{p_1}}$, then bidders are indifferent between purchasing tokens in period 1, not using them to bid, having them multiply by $1+\tau$, and then selling them in period 2. I call the demand for tokens that are not used for bidding the *speculative demand for tokens*, denoted by $S \geq 0$, which must be such that $p_1 = \frac{\sum_i \beta_{i,1}}{M-S} = (1+\tau)p_2^e$, or

$$\frac{\sum_i \beta_{i,1}}{M-S} = \frac{k_2}{M+\sigma(M-S)}$$

$$S = M\left(\frac{k_2 - (1+\sigma)\sum_i \beta_{i,1}}{k_2 - \sigma\sum_i \beta_{i,1}}\right)$$

Hence, there is an equilibrium with $p_1 = \frac{k_2}{M}$ if and only if $k_2 \geq (1+\sigma)\sum_i^n \beta_{i,1}$. In

[13] Clearly, there cannot be an equilibrium in which $p_1 < \frac{k_2}{M}$: if this was the case, bidders and the auctioneer can make unlimited profits by purchasing tokens in period 1 and selling them in period 2, and their demand for tokens is undefined.

such equilibrium, for given profile of bids, bidder i's two-periods payoff is

$$\begin{cases} v_i - \beta_{i,1} + w_t & \text{if } b_{i,1} > \max_{j \neq i} \{b_{j,1}\} \\ -\beta_{i,1} + w_t & \text{otherwise.} \end{cases}$$

where I used the fact that, by assumption, $a_{i,1} = 0$.

If instead $p_1 > (1+\tau)p_2^e = \frac{k_2}{M + \sigma \frac{\sum \beta_{i,1}}{p_1}}$, then, again, holding more tokens than what is necessary in order to bid generates a cost, because the future expected price of tokens is lower than the current price. Hence, again, $\beta_{i,1} = p_1(a_{i,1} + q_{i,1})$ and $\sum_i \beta_{i,1} = p_1 M$. This is an equilibrium if and only if

$$\frac{\sum_i \beta_{i,1}}{M} > \frac{k_2}{M(1+\sigma)}$$

or $(1+\sigma)\sum_i \beta_{i,1} > k_2$. In this equilibrium, using the fact that $a_{i,2} = 0$, for given profile of bids, bidder i's two-periods payoff as

$$\begin{cases} v_{i,1} - \beta_{i,1} + w_2 & \text{if } b_{i,1} > \max_{j \neq i} \{b_{j,1}\} \\ -\beta_{i,1} + w_2 & \text{otherwise.} \end{cases}$$

which is identical to the previous case. □

References

Bakos, Y. and H. Halaburda (2018). The role of cryptographic tokens and icos in fostering platform adoption. *working paper*.

Bakos, Y. and H. Halaburda (2019). Funding new ventures with digital tokens: due diligence and token tradability. *NYU Stern School of Business*.

Bulow, J. and P. Klemperer (1996). Auctions versus negotiations. *The American Economic Review 86*(1), 180–194.

Bulow, J. and J. Roberts (1989). The simple economics of optimal auctions. *Journal of political economy 97*(5), 1060–1090.

Canidio, A. (2018). Financial incentives for the development of blockchain-based platforms.

Canidio, A. (2020). Cryptotokens and cryptocurrencies: the extensive margin. Technical report, working paper.

Catalini, C. and J. S. Gans (2018, March). Initial Coin Offerings and the Value of Crypto Tokens. NBER Working Papers 24418, National Bureau of Economic Research, Inc.

Chod, J. and E. Lyandres (2021). A theory of icos: Diversification, agency, and information asymmetry. *Management Science 0*(0), null.

Cong, L. W., Y. Li, and N. Wang (2020). Token-based platform finance. Technical report, National Bureau of Economic Research.

Cong, L. W., Y. Li, and N. Wang (2021). Tokenomics: Dynamic adoption and valuation. *The Review of Financial Studies 34*(3), 1105–1155.

Gans, J. S. (2019). The fine print in smart contracts. Technical report, National Bureau of Economic Research.

Garratt, R. J. and M. R. van Oordt (2021). Entrepreneurial incentives and the role of initial coin offerings. *Journal of Economic Dynamics and Control*, 104171.

Goldstein, I., D. Gupta, and R. Sverchkov (2019). Initial coin offerings as a commitment to competition. *Available at SSRN 3484627*.

Gryglewicz, S., S. Mayer, and E. Morellec (2021). Optimal financing with tokens. *Journal of Financial Economics*.

Holden, R. T. and A. Malani (2019). Can blockchain solve the hold-up problem in contracts? Technical report, National Bureau of Economic Research.

Klemperer, P. (1999). Auction theory: A guide to the literature. *Journal of economic surveys 13*(3), 227–286.

Kocherlakota, N. R. (1998). Money is memory. *journal of economic theory 81*(2), 232–251.

Lee, M., A. Martin, and R. M. Townsend (2021). Optimal design of tokenized markets. *Available at SSRN 3820973*.

Li, J. and W. Mann (2018). Digital tokens and platform building. *Working paper*.

Lucas Jr, R. E. and N. L. Stokey (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of monetary Economics 12*(1), 55–93.

Malinova, K. and A. Park (2018). Tokenomics: when tokens beat equity. *Available at SSRN 3286825*.

Milgrom, P. and I. Segal (2002). Envelope theorems for arbitrary choice sets. *Econometrica 70*(2), 583–601.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of operations research 6*(1), 58–73.

Ostroy, J. M. and R. M. Starr (1974). Money and the decentralization of exchange. *Econometrica: Journal of the Econometric Society*, 1093–1113.

Riley, J. G. and W. F. Samuelson (1981). Optimal auctions. *The American Economic Review 71*(3), 381–392.

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of political economy 66*(6), 467–482.

Sockin, M. and W. Xiong (2018). A model of cryptocurrencies. *Working paper*.

Starr, R. M. (1974). The price of money in a pure exchange monetary economy with taxation. *Econometrica 42*(1), 45–54.

Townsend, R. M. (1980). Models of money with spatially separated agents. *Models of monetary economies*, 265–303.

Townsend, R. M. (2020). *Distributed Ledgers: Design and Regulation of Financial Infrastructure and Payment Systems.* MIT Press.

Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance 16*(1), 8–37.