The Transition- Dynamics from Government Money only to Differentiated (Crypto-) Currency Competition

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Motivation

Why do people need and hold money?

Menger (1871), Mann (1971), Hayek (1990), Berentsen & Schär (2018)

- Cash purchases, holding reserves for future payments, standard of deferred payments, reliable unit of account
- Transaction function - facilitates trading

Is currency better supplied by monopoly or competition?

Friedman (1960), Hayek (1990), White (2015)

- Benevolence vs. profit-maximization

How do people adapt an (alternative) currency?

Kiyotaki & Wright (1993), Luther (2016), Alzahrani & Daim (2019)

- Network effect
- Changes in money holdings and behaviour

→ **State and analyse transition dynamics in a New Monetarist Model**
Lagos & Wright (2003, 2005), Williamson & Wright (2010), Waknis (2017), Fernández-Villaverde & Sanches (2016)

Specifics of Cryptocurrencies

- Number of Altcoins rose rapidly
- Different characteristics
  - Anonymity
  - Transaction time
  - Transaction costs
  - Volatility/ Stability
  - …
- Wider functionality
  - Smart contracts (dapps, DeFi, tokenisation)
- Different reasons to hold and use a cryptocurrency
- Blockchain technology and cryptocurrencies keep improving

A New Monetarist Model
The Life of the Agents

[0,1]-continuum of agents
discrete time

Agents decide how much to **work**, **trade** & **consume** to maximize utility with discount factor $\beta \in (0,1)$

Day-Market DM

period t

Night-Market CM

- Decentralized Market
- Random bilateral matching
- Specialized good q

$$\begin{aligned}V(\boldsymbol{m_t}, \boldsymbol{s_t}) =& \alpha\sigma\boldsymbol{\mu_t}\psi_t \int \{u[q(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})] + W[\boldsymbol{m_t} - \boldsymbol{d_t}(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})]\}\mathrm{d}F(\boldsymbol{\tilde{m}_t})\\ &+\alpha\sigma\boldsymbol{\mu_t}\psi_t \int \{-c[q(\boldsymbol{\tilde{m}_t}, \boldsymbol{m_t}, \boldsymbol{s_t})] + W[\boldsymbol{\tilde{m}_t} + \boldsymbol{d}(\boldsymbol{\tilde{m}_t}, \boldsymbol{m_t}, \boldsymbol{s_t})]\}\mathrm{d}F(\boldsymbol{\tilde{m}_t})\\ &+\alpha\delta \int B(\boldsymbol{m_t}, \boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})\mathrm{d}F(\boldsymbol{\tilde{m}_t})\\ &+(1 - 2\alpha\sigma\boldsymbol{\mu_t}\psi_t - \alpha\delta)W(\boldsymbol{m_t}, \boldsymbol{s_t})\end{aligned}$$

- Centralized Market
- General good x

$$W(\boldsymbol{m_t}, \boldsymbol{s_t}) = \max_{x,H,\boldsymbol{m_{t+1}}} \{U(x) - AH + \beta V(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})\}$$

$$s.t.\ \ x = \omega H + \boldsymbol{\phi_t m_t} + \boldsymbol{y_t m_t} - \tau - \boldsymbol{\phi_t m_{t+1}}$$

A New Monetarist Model
The Role of Money



The use of money is not necessary but may enhance trading!

Day-Market DM | period t | Night-Market CM

- **Single-coincidence meeting**:
  pay/ receive money for specialized good q
- **Double-coincidence meeting**:
  barter trade

- Pay for or barter trade the general good x

- Agents decide how much money they want to hold and save for the next period $\boldsymbol{m_{t+1}}$
- Optimal portfolio choice equilibrium condition:

$$\beta(\phi^i_{t+1} + y^i_{t+1})\{1 + l^i[q_{t+1}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})]\} = \phi^i_t$$

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \equiv \alpha\sigma\mu^i_t(\frac{u'[q_t(\boldsymbol{m_t}, \boldsymbol{s_t})]}{c'[q_t(\boldsymbol{m_t}, \boldsymbol{s_t})]} - 1)$$

ϕ^i_t… value of currency i at time t
y^i_t… additional-value of currency i at time t
l^i_t… liquidity premium of currency i at time t

A New Monetarist Model
Equilibria

Only one Government Currency

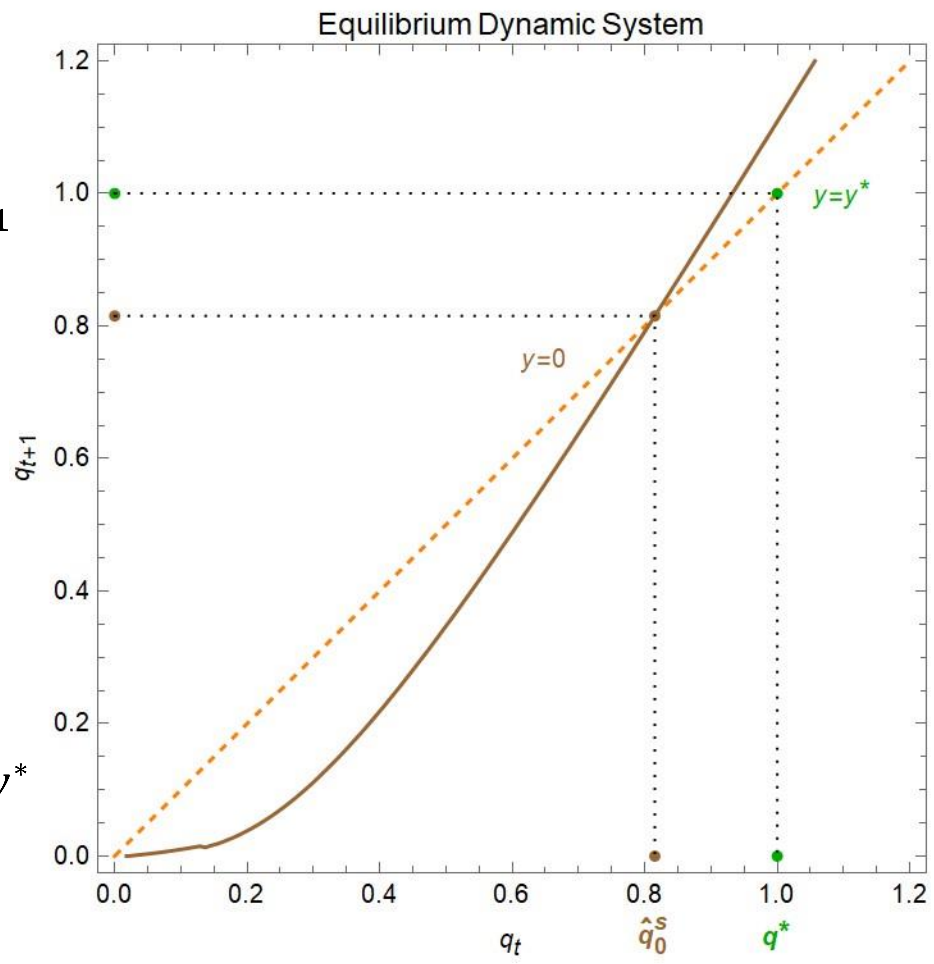
$$\beta\phi^g_{t+1}\{1 + l^g[\hat{q}_{t+1}(m_{t+1}, \boldsymbol{s_{t+1}})]\} = \phi^g_t$$

- Steady state trading output $\hat{q}^s_0$ with price stability $\phi^g_t = \phi^g_{t+1}$
- BUT: trading activity lower than socially optimal
- BUT: equilibrium trajectory with declining trading activity and declining money value, i.e. inflation

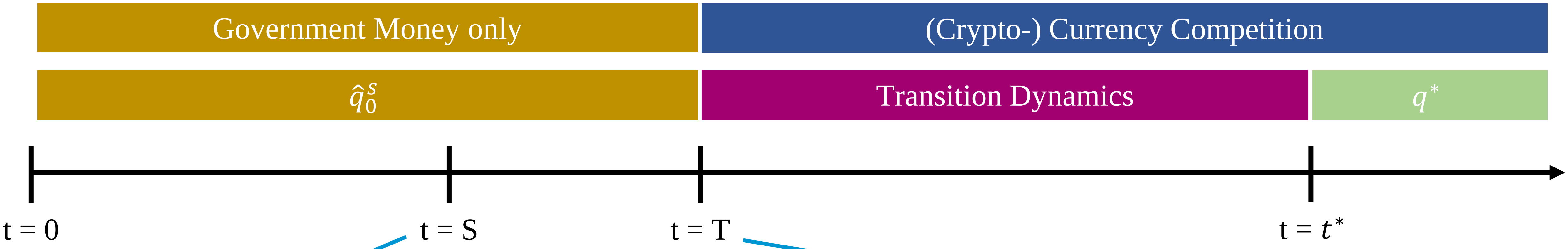
(Crypto-) Currency Competition

$$\beta(\phi^i + y^*)\{1 + l^i[q^*(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})]\} = \phi^i$$

- Unique steady state socially optimum trading output $\boldsymbol{q^*}$
- Cryptocurrencies provide maximum feasible additional value y^*
- Cryptocurrencies stable value $\phi^c_t = \phi^c_{t+1}$
- Government currency stable deflation rate $\rho^* = \dfrac{\phi^g_{t+1}}{\phi^g_t} > 1$



A New Monetarist Model
Evolving Competition



t= S: Cryptocurrency becomes available in the economy

- Low initial acceptability
  $\mu_S^c \ll \mu^g = 1$
- Network effect prevents circulation in equilibrium
- But not perfect substitutes

Different characteristics and additional services can compensate lower acceptability

t = T: Cryptocurrency starts circulating in equilibrium

- The lower the initial acceptability, the higher the necessary additional value for the agents

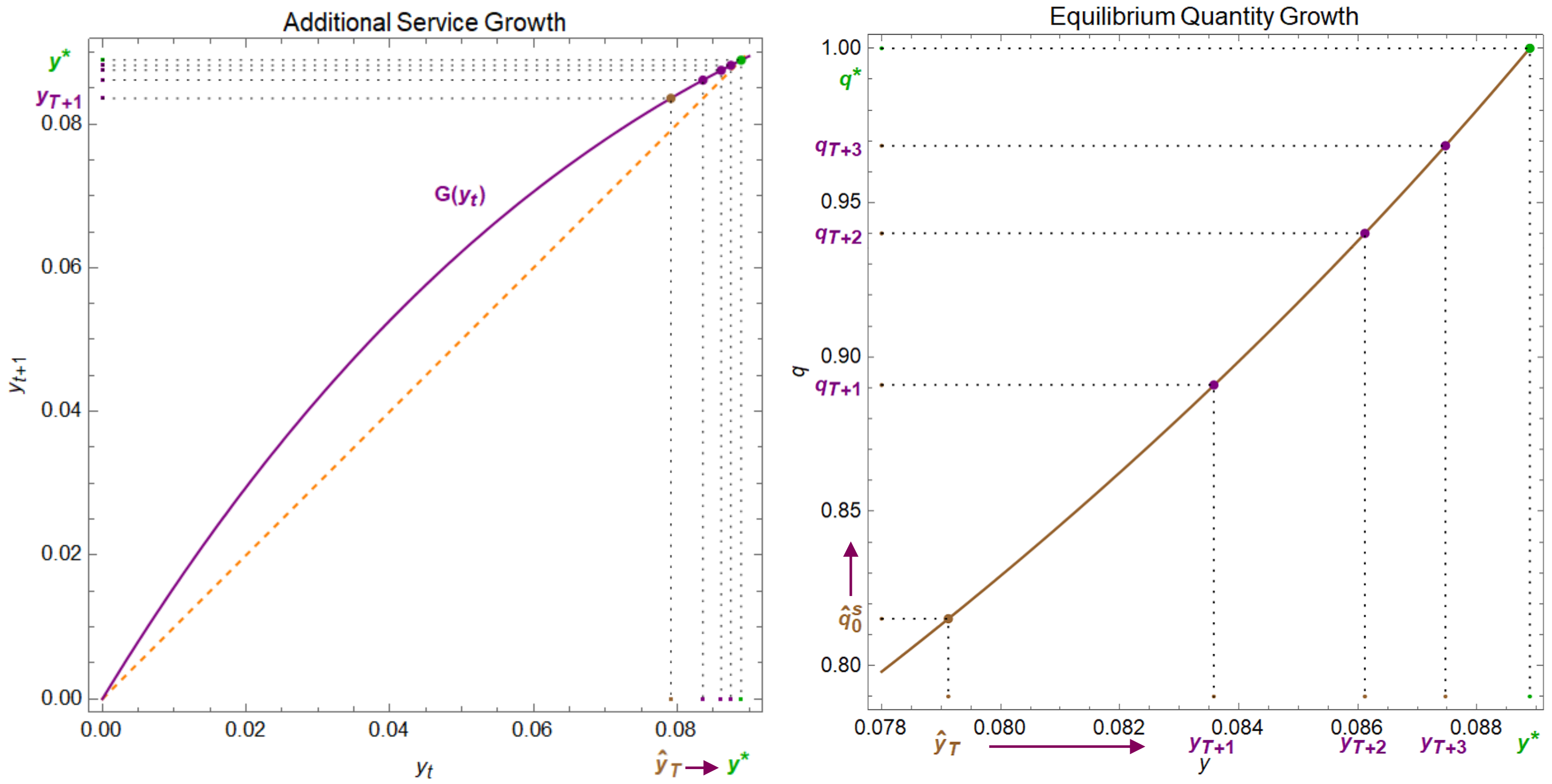
A New Monetarist Model
Transition Dynamics

Technology improves

→ Additional service grows

$$n(y_t^c) = \frac{y_{t+1}^c - y_t^c}{y_t^c} = \frac{G(y_t^c)}{y_t^c} - 1 \geqslant 0 \;\; \forall t \geqslant S$$
$$G(y_t^c) \geqslant y_t^c > 0 \;\; \forall y_t^c \leqslant y^* \;\; (positive)$$
$$n(y_t^c) \geqslant n(y_{t+1}^c) \geqslant 0 \;\; (decreasing)$$
$$\lim_{t\to+\infty} n(y_t^c) = 0 \;\; (tends\ to\ zero)$$
$$\exists y_\infty^* \;\; s.t. \;\; y_t \leqslant y_\infty^* \;\; \forall t \;\; (bounded)$$

→ Trading output increases



A New Monetarist Model
Transition Dynamics II

$\boldsymbol{n(y_t^c)}$ $\qquad\qquad\qquad\qquad$ $\boldsymbol{g(q_t)}$

Additional Service $y_t^c \uparrow$ → Agent's Money holdings $\boldsymbol{m_t} \uparrow$ → Traded Quantity $q_t \uparrow$ → Government Money Rate of Return $\rho_t^g \uparrow$

According to these dynamics the economy converges to the first-best unique steady state equilibrium:

- Cryptocurrency performs the best it can $\boldsymbol{y}^*$
- Cryptocurrency is stable $\phi_t^c = \phi_{t+1}^c$
- Socially optimum trading output $\boldsymbol{q}^*$
- Government currency stable deflation rate $\rho^* = \frac{\phi_{t+1}^g}{\phi_t^g} > 1$ (Friedman rule on currency)

A New Monetarist Model
Various Different Competing Cryptocurrencies

- Fundamental dynamics and properties persist
- Free-entry and market clearing condition need to be fulfilled
- Need to provide enough additional value to compensate different lower acceptability
  - The more they are accepted, the lower is the necessary additional value to start circulating
  - If not sufficient, they do not get valued in economy equilibrium
- → Some cryptocurrencies may fail and lose their business in the perfect competitive currency environment

- Same *amount* of additional value does not imply same additional value
- Circulating cryptocurrencies can differ in their characteristics and services provided

- In first-best equilibrium with y^* all cryptocurrencies perform the best they can

A New Monetarist Model
Growing Acceptability

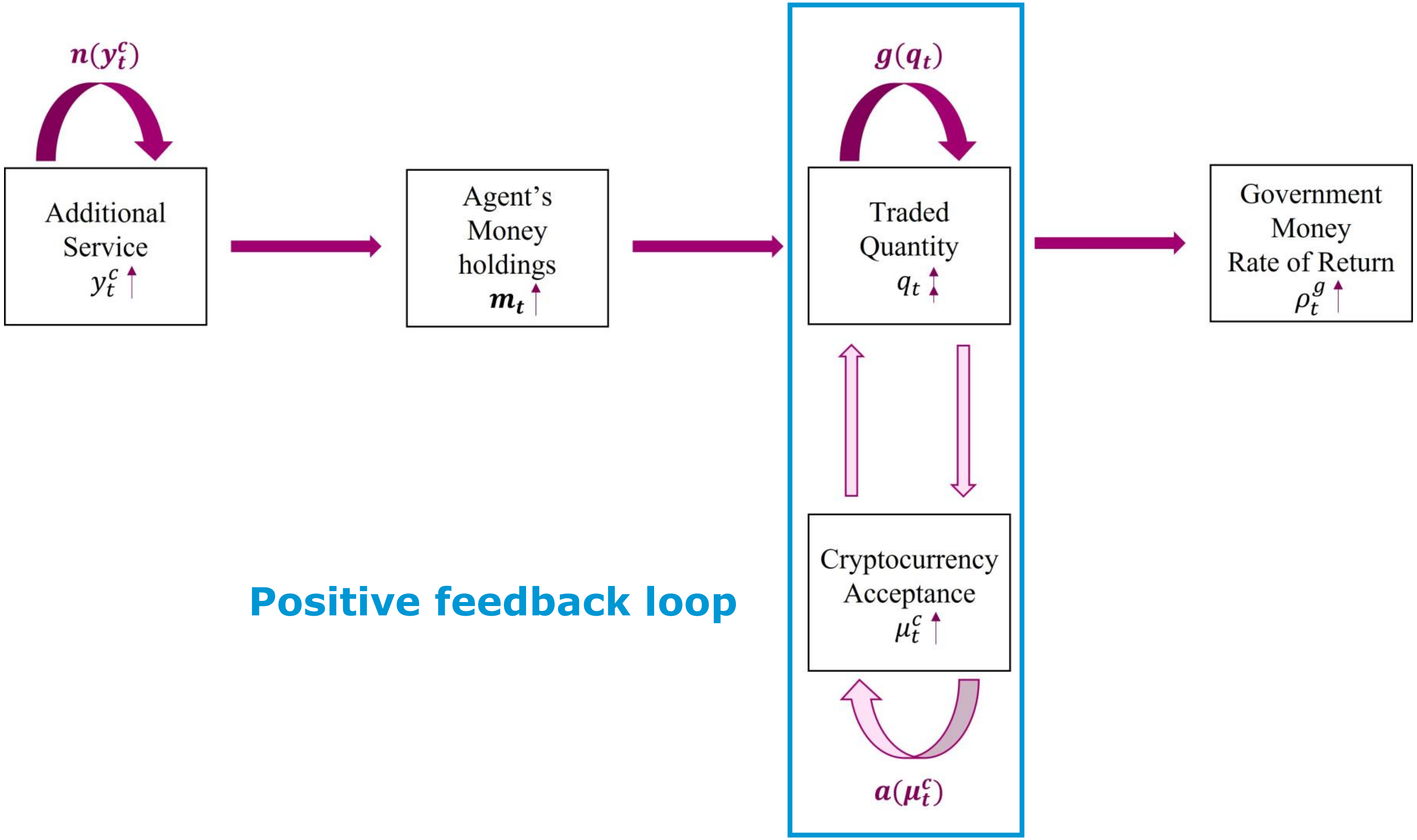
- Assume agents observe the enhanced trading activity through cryptocurrencies
  → Learn about new possibilities
  → More agents decide to accept this currency

- Assume acceptability of this currency increases related to output growth with one period lag

$$g(q_t) = \frac{q_{t+1} - q_t}{q_t} = \frac{q_{t+1}}{q_t} - 1 \geqslant 0 \;\; \forall t \geqslant T$$

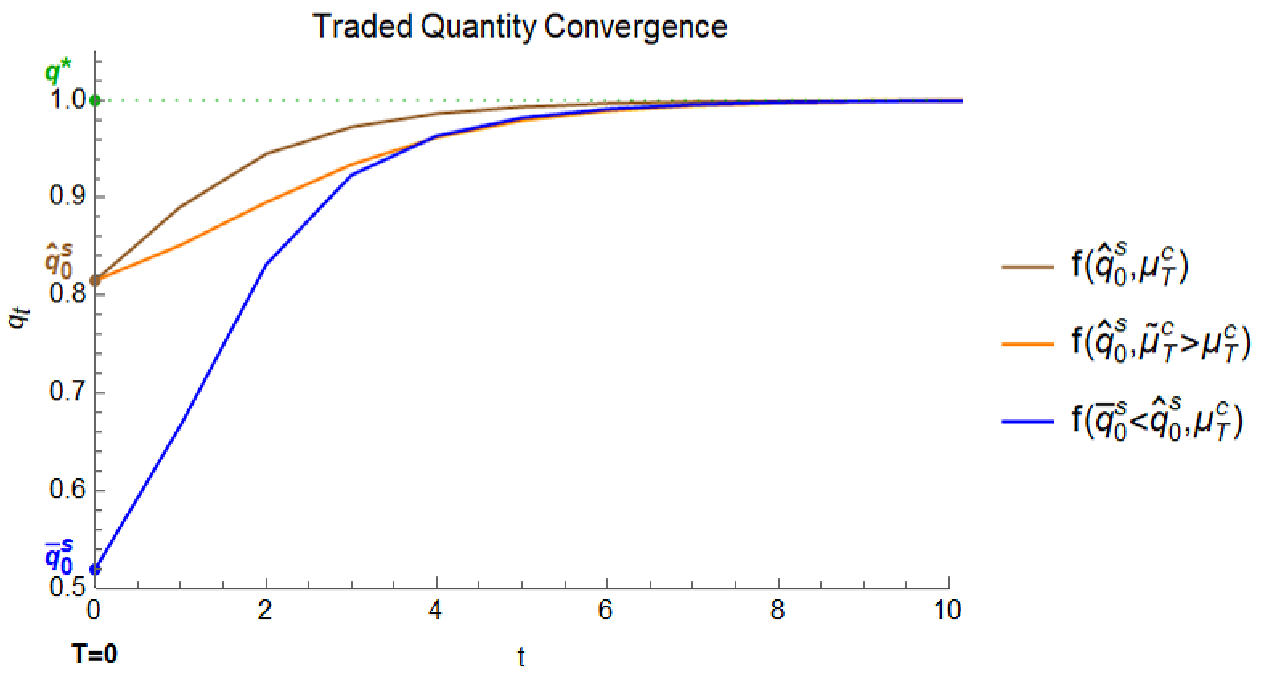
$$a(\mu_t^c) = \frac{\mu_{t+1}^c - \mu_t^c}{\mu_t^c} \equiv A[g(q_{t-1})] \geqslant 0 \;\; \forall t \geqslant T + 1$$

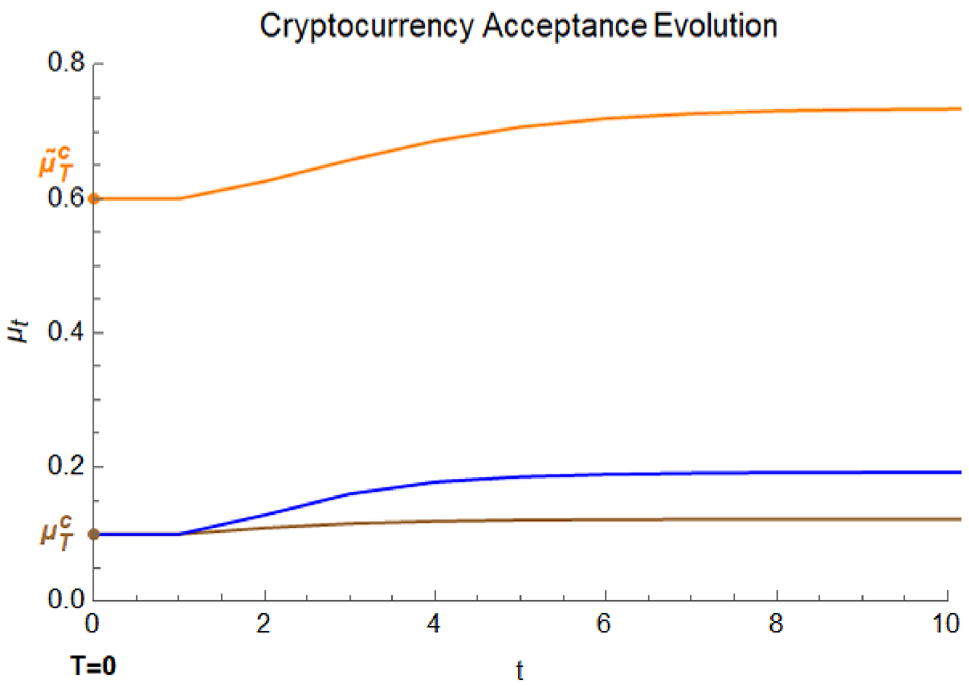
A New Monetarist Model
Transition Dynamics III



Positive feedback loop

A New Monetarist Model
Simulation





- Overall, economically struggling economies gain more from (crypto-) currency competition
- Overall, cryptocurrency acceptability does not rise that much
- Cryptocurrencies establish themselves as various niche monies with different characteristics

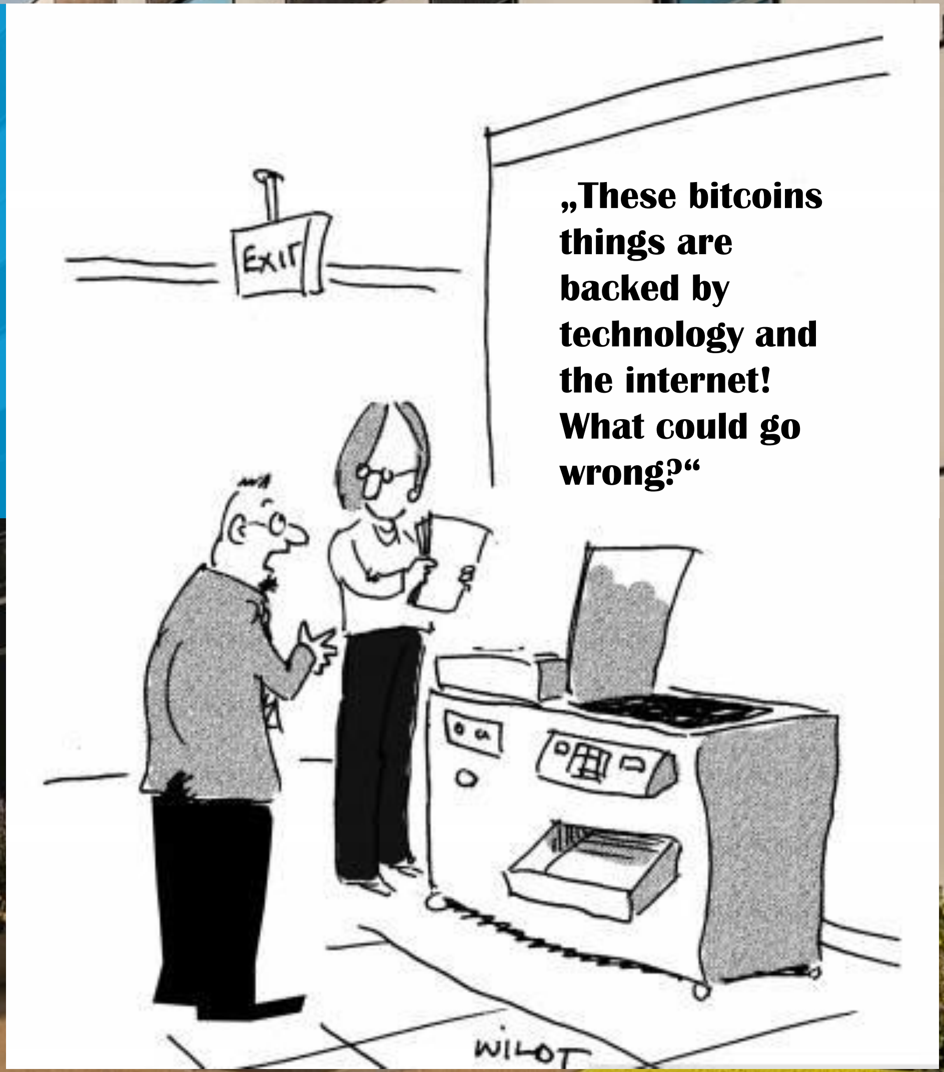
Conclusion

- Economy can profit from differentiated (crypto-) currency competition through increased money holdings and enhanced trading activity
- Main drivers for cryptocurrency adoption and economy transition are the different characteristics and value-added services of cryptocurrencies and technology growth to overcome strong network effects
- Cryptocurrencies establish themselves as various niche monies serving different consumer's needs
- (Crypto-) currency competition can lead to cryptocurrency price stability and a stable government money deflation rate

BUT:

- Model builds upon restrictive assumptions and simplifications
- Deflation generally not seen as desirable outcome for monetary policy
- Systemic and network risks in highly interconnected systems
- Further research needed

Thank you for your attention!



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Appendix

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Appendix

- Quasi-linear utility function of the agents in the economy

$$\mathcal{U}(q, x, H) = u(q) - c(q) + U(x) - AH$$

Furthermore, we assume that $u(q)$, $c(q)$ and $U(x)$ are C^n with $n \geqslant 2$, $u^{'} > 0$, $u^{''} < 0$, $c^{'} > 0$, $c^{''} \geqslant 0$, $u(0) = c(0) = 0$, $U^{'} > 0$, $U^{''} \leqslant 0$ and there exists $q^* \in (0, \infty)$ $s.t.$ $u^{'}(q^*) = c^{'}(q^*)$ as well as a $x^* \in (0, \infty)$ $s.t.$ $U^{'}(x^*) = 1$ and $U(x^*) > x^*$.

- Dummy variable vor agent's currency holding

$$\psi_t^i = \begin{cases} 1 & \text{if } m_t^i > 0 \\ 0 & \text{if } m_t^i = 0 \end{cases}$$

Appendix
Single-coincidence meeting

Centralized Nash bargaining problem, buyer has full bargaining power $\theta = 1$

$$\max_{q,\boldsymbol{d}} [u(q) + W(\boldsymbol{m} - \boldsymbol{d}, \boldsymbol{s}) - W(\boldsymbol{m},\boldsymbol{s})]^{\theta}[-c(q) + W(\tilde{\boldsymbol{m}} + \boldsymbol{d}, \boldsymbol{s}) - W(\tilde{\boldsymbol{m}}, \boldsymbol{s})]^{1-\theta}$$

$$s.t. \;\; \boldsymbol{d} \leqslant \boldsymbol{m}$$

$$\max_{q,\boldsymbol{d}} [u(q) - (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d}]$$

$$s.t. \;\; \boldsymbol{d} \leqslant \boldsymbol{m}$$

$$s.t. \quad -c(q) + (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{d} \geqslant 0$$

$$q(\boldsymbol{m_t}, \boldsymbol{s_t}) = \begin{cases} \hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t}) & \text{if } \; (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ q^* & \text{if } \; (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} \geqslant c(q^*) \end{cases}$$

$$(\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{d_t}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \begin{cases} (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} & \text{if } \; (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ c(q^*) & \text{if } \; (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} \geqslant c(q^*) \end{cases}$$

Appendix
Double-coincidence meeting

Symmetric Nash bargaining problem

$$\max_{q_1,q_2,\boldsymbol{\Delta}} [u(q_1) - c(q_2) - (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{\Delta}][u(q_2) - c(q_1) + (\boldsymbol{\phi} + \boldsymbol{y})\boldsymbol{\Delta}]$$

$$s.t. \quad -\boldsymbol{m_2} \leqslant \boldsymbol{\Delta} \leqslant \boldsymbol{m_1}$$

$$\begin{aligned} \boldsymbol{\Delta} = \lambda_1 = \lambda_2 &= 0 \\ q_1 = q_2 &= q^* \\ u'(q^*) &= c'(q^*) \\ B(\boldsymbol{m}, \tilde{\boldsymbol{m}}, \boldsymbol{s}) &= u(q^*) - c(q^*) + W(\boldsymbol{m}, \boldsymbol{s}) \end{aligned}$$

Appendix
Bellman‘s equation

$$
\begin{aligned}
V(\boldsymbol{m_t}, \boldsymbol{s_t}) = &\max_{\boldsymbol{m_{+1}}}\{-\boldsymbol{\phi_t m_{t+1}} + \beta V(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})\} \\
&+ U(x^*) - x^* + \nu(\boldsymbol{m_t}, \boldsymbol{s_t}) \\
&+ (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} - \tau
\end{aligned}
$$

$$
\nu(\boldsymbol{m_t}, \boldsymbol{s_t}) = \begin{cases}
\quad \alpha\sigma\boldsymbol{\mu_t}\boldsymbol{\psi_t}\{u[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})] - (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t}\} & \\
+ \quad \alpha\sigma\boldsymbol{\mu_t}\boldsymbol{\psi_t} \int \{-c[\hat{q}(\boldsymbol{\tilde{m}_t}, \boldsymbol{s_t})] + (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{\tilde{m}_t}\} \mathrm{d}F(\boldsymbol{\tilde{m}}) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\
+ \quad \alpha\delta\{u(q^*) - c(q^*)\} & \\
& \\
\quad \alpha(\sigma\boldsymbol{\mu_t}\boldsymbol{\psi_t} + \delta)\{u(q^*) - c(q^*)\} & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} \geqslant c(q^*)
\end{cases}
$$

Appendix
Optimal portfolio choice

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_{t+1}} = -\phi^i_t + \beta V^{i'}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}}) \leqslant 0$$

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_t} = \begin{cases} \alpha\sigma\mu^i_t(\phi^i_t + y^i_t)\frac{u^{i'}[\hat{q}(\boldsymbol{m_t},\boldsymbol{s_t})]}{c^{i'}[\hat{q}(\boldsymbol{m_t},\boldsymbol{s_t})]} + (1-\alpha\sigma\mu^i_t)(\phi^i_t + y^i_t) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ (\phi^i_t + y^i_t) & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \equiv \alpha\sigma\mu^i_t(\frac{u'[\hat{q}_t(\boldsymbol{m_t}, \boldsymbol{s_t})]}{c'[\hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t})]} - 1)$$

$$\frac{\partial V(\boldsymbol{m_t}, \boldsymbol{s_t})}{\partial m^i_t} = (\phi^i_t + y^i_t)\{1 + l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})]\}$$

$$\beta(\phi^i_{t+1} + y^i_{t+1})\{1 + l^i[q_{t+1}(\boldsymbol{m_{t+1}}, \boldsymbol{s_{t+1}})]\} = \phi^i_t$$

$$l^i[q(\boldsymbol{m_t}, \boldsymbol{s_t})] \begin{cases} \neq 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} < c(q^*) \\ = 0 & \text{if } (\boldsymbol{\phi_t} + \boldsymbol{y_t})\boldsymbol{m_t} > c(q^*) \end{cases}$$

Appendix
Government money

$$b_t^g \equiv \phi_t^g M_t^g$$

budget constraint yields

$$\phi_t^g M_t^g + \tau_t = \phi_t^g M_{t-1}^g$$

return on government money is

$$\rho_t^g \equiv \frac{\phi_t^g}{\phi_{t-1}^g}$$

Appendix
Private currency issuers

$$\sum_{t=S}^{\infty} \beta^t x_t^c$$

$$x_t^c = \phi_t^c \Delta_{M_t^c} + y_t^c \Delta_{M_t^c} - \sum_{i \neq c} \phi_t^i \Delta_{M_t^i} - \sum_{i \neq c} y_t^i \Delta_{M_t^i}$$

$$b_t^c \equiv (\phi_t^c + y_t^c) M_t^c$$

Appendix
Money equilibrium

- Money supply

$$\sum_{i=1}^{N} b_t^i = b_t^g + \sum_{c=1}^{C} b_t^c = \phi_t^g M_t^g + (\phi_t^c + y_t^c) M_t^c \;\; \forall t$$

- Money demand

$$\sum_{i=1}^{N} b_t^i = z[\hat{q}_t(\boldsymbol{m_t}, \boldsymbol{s_t})]$$

Appendix
Equilibria

Definition 1. *Any equilibrium satisfies (12) and (19) with the boundary condition* $\beta(\phi^i_{t+1} + y^i_{t+1}) \leqslant \phi^i_t$ *and* $b^i_t \geqslant 0\ \forall i \in \{1, ..., N\}$ *where* $q(\boldsymbol{m_t}, \boldsymbol{s_t}) = \hat{q}(\boldsymbol{m_t}, \boldsymbol{s_t}) \leqslant q^*$ *and* $\mathbf{d}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \boldsymbol{m_t} \leqslant \boldsymbol{m^*_t}$ *with* $\boldsymbol{m^*_t} \equiv \frac{c(q^*)}{(\boldsymbol{\phi_t}+\boldsymbol{y_t})}$ *at all time subject to the monetary policy the government implements and the optimal behaviour of the cryptocurrency issuers.*

Proposition 1. *An equilibrium with only one government money in circulation yields a monetary steady state equilibrium* $0 < \hat{q}^s < q^*$ *delivering price stability* $(\rho^g, \rho^g_{+1}) = (1, 1)$ *as well as a nonmonetary steady state* $(\rho^g, \rho^g_{+1}) = (0, 0)$ *with* $\hat{q} = 0$. *Furthermore there exists an equilibrium trajectory where* $\hat{q}_t \to 0\ \ \forall \hat{q}_0 \in (0, q^*)$.

Proposition 2. *There exist a unique steady state equilibrium with circulating competitive currencies and the socially optimum trading activity* q^* *if the cryptocurrencies perform the best they can and implement* $y^* = \frac{(1-\beta)}{\beta}\phi^c$ *with a stable value* $\phi^c_t = \phi^c_{t+1} = \phi^c$ *and the government currency provides a stable deflation rate* $\rho^g > 1\ \ \forall t$ *satisfying the equilibrium condition (12), the market-clearing condition (19) and the boundary condition* $\beta(\phi^i_{t+1}+y^i_{t+1}) \leqslant \phi^i_t$. *Thus,* $\mathbf{d}(\boldsymbol{m_t}, \boldsymbol{s_t}) = \boldsymbol{m_t} = \boldsymbol{m^*_t}$ *with* $\boldsymbol{m^*_t} \equiv \frac{c(q^*)}{(\boldsymbol{\phi_t}+\boldsymbol{y_t})}$.

Appendix
Functional forms

$$\{1+\alpha\sigma\mu_{t+1}^{i}(\frac{u'[q_{t+1}(\boldsymbol{m_{t+1}},\boldsymbol{s_{t+1}})]}{c'[q_{t+1}(\boldsymbol{m_{t+1}},\boldsymbol{s_{t+1}})]}-1)\}=\frac{\phi_t^i}{\beta(\phi_{t+1}^i+y_{t+1}^i)}$$

$$u(q)=(1-\eta)^{-1}q^{1-\eta} \qquad c(q)=(1+\gamma)^{-1}q^{1+\gamma}$$

$0<\eta<1 \quad \gamma\geq 0$

$$\{1+\alpha\sigma\mu_{t+1}^{i}(q_{t+1}^{-(\gamma+\eta)}-1)\}=\frac{\phi_t^i}{\beta(\phi_{t+1}^i+y_{t+1}^i)}$$

$$\{1+\alpha\sigma\mu^{c}(\hat{q}_0^{s\,-(\gamma+\eta)}-1)\}=\frac{\phi^c}{\beta(\phi^c+\hat{y}_T)}\equiv\{1+\alpha\sigma\mu^{g}(\hat{q}_0^{s\,-(\gamma+\eta)}-1)\}=\frac{\phi^g}{\beta\phi^g}$$

Appendix
Transition dynamics

Time T+1. *At point $T+1$ in time we derive an equilibrium for the economy where the government currency and the cryptocurrency are circulating in a competitive environment with the traded quantity $q_{T+1} > \hat{q}{}_0^s$ due to an increase in the agents money holdings $\boldsymbol{m_{T+1}} > \boldsymbol{m_T}$ according to the increased additional service $y_{T+1} > \hat{y}_T$. The government currency is fully accepted $\mu^g = 1$ but deflates $\rho^g_{T+1} > 1$ and the cryptocurrency still provides stability $\phi^c_{T+1} = \phi^c_T$ and circulates with acceptability $\mu^c << \mu^g$.*

Time $\boldsymbol{t^*}$. *At time t^* the economy reaches the first-best equilibrium with $q^* > \hat{q}{}_0^s$, $y^* > \hat{y}_T$, $\boldsymbol{m^*} > \boldsymbol{m_T}$ and $\mu^c << \mu^g = 1\ \forall t \geqslant t^*$. The cryptocurrency is stable $\phi^c_{t*} = \phi^c_{t*+1}$ and the government currency provides a stable deflation rate $\rho^* > \rho^g = 1$. $n(y^c_{t^*}) = g(q_{t^*}) = 0$ and the economy is expected to stay in this steady state forever.*

Appendix
Acceptability growth

Time T+2. *In period* $T+2$ *the acceptability of the cryptocurrency increases* $\mu^c_{T+2} > \mu^c_{T+1} = \mu^c_T$ *due to the observed increase in the traded quantity the period before* $a(\mu^c_{T+1}) \equiv A[g(q_T)]$. *Again the technology improves resulting in an higher additional service provided by the cryptocurrency* $y_{T+2} > y_{T+1}$. *Hence the overall trading output enlarges even further* $q_{T+2} > q_{T+1}$. *The cryptocurrency is still stable* $\phi^c_{T+2} = \phi^c_{T+1}$ *while the deflation rate of the government money rises* $\rho^g_{T+2} > \rho^g_{T+1} > 1$.

Time $\boldsymbol{t^*}$. *At time* t^* *the economy reaches the first-best equilibrium with* $q^* > \hat{q}^s_0$, $y^* > \hat{y}_T$, $\boldsymbol{m^*} > \boldsymbol{m_T}$, $\mu^g = 1$ *and* $1 \geqslant \mu^c_{t^*} > \mu^c_T$. *The cryptocurrency is stable* $\phi^c_{t^*} = \phi^c_{t^*+1}$ *and the government currency provides a stable deflation rate* $\rho^* > \rho^g = 1$. $n(y^c_{t^*}) = g(q_{t^*}) = a(\mu^c_{t^*}) = 0$ *and the economy is expected to stay in this steady state forever.*

Appendix
Acceptability growth